5 a) $|n\rangle = \frac{1}{\sqrt{n!}}(a^\dagger)^n|0\rangle$, $a|n\rangle = \sqrt{n}\,|n-1\rangle$

$H = \hbar\omega\left(\underbrace{a^\dagger a}_{}+\frac{1}{2}\right)$

Vernichtungsoperator

$$a\exp(\lambda a^\dagger)|0\rangle = a\sum_{n=0}^{\infty}\frac{\lambda^n (a^\dagger)^n}{n!}|0\rangle = a\sum_{n=0}^{\infty}\lambda^n\frac{1}{\sqrt{n!}}|n\rangle$$

$$=\lambda\sum_{n=0}^{\infty}\lambda^{n-1}\frac{1}{\sqrt{n!}}\cdot\sqrt{n}\,|n-1\rangle = \lambda\sum_{n=1}^{\infty}\lambda^{n-1}\frac{1}{\sqrt{(n-1)!}}|n-1\rangle$$

$$=\lambda\sum_{n=0}^{\infty}\lambda^n\frac{1}{\sqrt{n!}}|n\rangle = \lambda\sum_{n=0}^{\infty}\frac{\lambda^n}{n!}(a^\dagger)^n|\ldots\rangle = \lambda|\lambda\rangle$$

b) $\langle 0|\exp(\lambda a^\dagger)^\dagger\exp(\lambda a^\dagger)|0\rangle = \sum_n\frac{|\lambda|^{2n}}{n!} = \exp(|\lambda|^2)$

$$=\frac{1}{\sqrt{(\exp|\lambda|^2)}} = \exp\left(-\frac{1}{2}|\lambda|^2\right) = \text{Normierungskonstante}$$

6 a) $H = \frac{p^2}{2m}$

$$\frac{dx_H}{dt} = \frac{1}{i\hbar}[x_H, H_H] = \frac{p_H(t)}{m}$$

$$\frac{dp_H}{dt} = \frac{1}{i\hbar}[p,H] = 0 \quad\Rightarrow\quad p_H(t) = p_0$$

b) $\langle p\rangle(t) = \langle p\rangle_0 \qquad \langle x\rangle(t) = \langle x\rangle_0 + \frac{\langle p\rangle_0}{m}t$ $\qquad (x_H(t))\int_{t=0}^{t}\frac{dx_H}{dt}dt'$

c) $[x_H(t_2), x_H(t_1)] = [x_s+\frac{p_s}{m}t_2, x_s+\frac{p_s}{m}t_1] = \frac{t_2}{m}[p_s, x_s] + \frac{t_1}{m}[x_s, p_s]$

$$=\frac{\hbar}{i}\,\frac{t_2-t_1}{m}$$

Unschärfe steigt mit der Unschärfe

$[p_H(t_2), p_H(t_1)] = 0$ da beide unabhängig von der Zeit

$[x_H(t_2), p_H(t_1)] = [x_s+\frac{p_s}{m}t, p_0] = [x_s, p] = i\hbar$